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Simple Feedback & It's Benefits

Introduction

Feedback is one of those extremely useful techniques used in everyday life. You may not know it, but everything you do has a similarity with how feedback is used in electrical circuits. When you reach out and touch the hot cookie sheet in the oven, a searing pain reports to your hand and brain causing it to retract. Without this *feedback*, you'd just continue to pick up the cookie sheet without any pot holders and discover some nasty burns where you had handled it. Similarly feedback is used in circuits to condition, or guide some output, given an input, to a desired result.

Basic Component of Feedback: the OpAmp

The OpAmp (operational amplifier) is the simplest component used in most analog feedback circuitry. It's not a necessary component, but circuits without OpAmps can be rewritten to use OpAmps (or at least the ideal OpAmp). The standard OpAmp is like the hand touching the hot cookie sheet. It's mostly a dual state device (okay or ouch!), which can be led to a third state using feedback.

Figures 1a and *1b* show how an ideal OpAmp can be pictured. *1a* shows how it is drawn in schematic form. *1b* is just a conceptual picture showing that it is like a two state switch which chooses between the values of positive infinity and negative infinity. The differential input is such that when the voltage potential is positive from '+' to '-', there is a positive infinite gain, and when the voltage potential is negative from '-' to '+', there is a negative infinite gain. By looking at *figure 1b*, you can see that the switch points in the direction of the highest potential. Other important features of the ideal OpAmp are infinite input impedance, infinitely fast switch times, and zero output impedance (perfect voltage source).

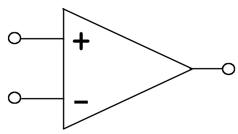

Fig 1a: The Ideal OpAmp

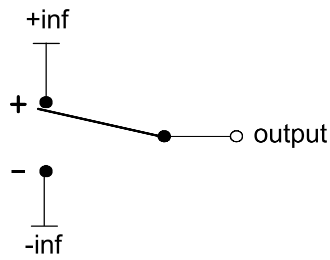

Fig 1b: The Dual State Representation

Guiding the OpAmp Output

The idea behind feedback is to guide the output of the OpAmp until it reaches the value that you desire. If the output is too high, you flip the switch so that the output decreases. If the output is too low, you flip the switch so that the output increases. If the output is just right, the switch is stuck in-between contacts and the output holds steady (or maybe it toggles infinitely fast between the two states, thus averaging zero change). This is like steering a car. If you are pointing away from the middle of the road, you point the wheels to move back towards the center. In this way, you stay at the middle of your lane.

So, what if we tried guiding the OpAmp by hand just like a car? Imagine that you can move at infinite speeds. *Figure 2* represent how you would control it. There is a device to read the output value of the OpAmp (voltage or current), you detect this value with your eyes, and then toggle the switch at the front of the OpAmp. The challenge with building feedback into circuits is to detect and move the switch without having to do it manually.

What do you do when the value climbs too high? You pull back to the negative terminal. What do you do when the value climbs too low? You push up to the positive terminal. A block diagram (shown in *figure 3*) of the system can be drawn to represent this. To model what is happening in *figure 2*, the gain of the feedback amplifier can be given as, and the input can represent the desired output. This way when the output drops below the input, the input of the feedforward amplifier (the differential input for the OpAmp is represented by the subtracting of one signal from the other) becomes negative, causing the output to drop. Likewise, if the output is greater than the input, the input of the feedforward amplifier becomes positive, causing the output to climb.

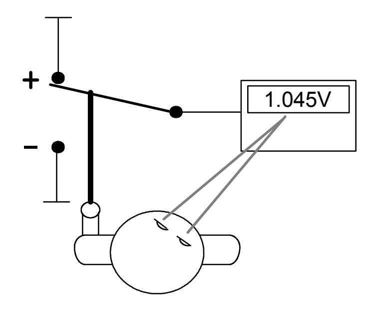

Fig 2: OpAmp Guiding

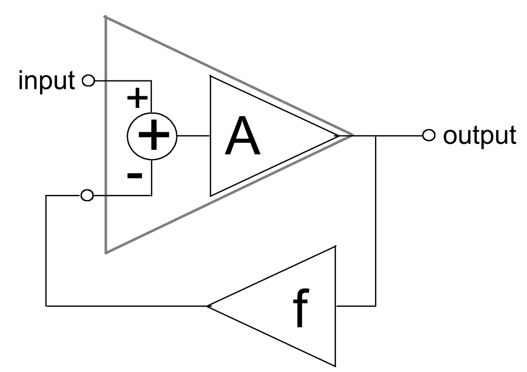

Fig 3: Feedback Block Diagram

What if $f \neq 1$? An equation can be developed and solved to find out what happens:

$$Out = A \cdot (In - f \cdot Out)$$

$$Out + A \cdot f \cdot Out = A \cdot In$$

$$Gain = \frac{Out}{In} = \frac{A}{1 + A \cdot f} = \frac{1}{\frac{1}{A} + f}$$

$$Gain = \lim_{A \to \infty} \frac{1}{\frac{1}{A} + f} = \frac{1}{f}$$

The gain approaches the inverse of the feedback gain! That would explain why $f = 1$ reproduces the input at the output. In order to get gain at the output, f must be less than one. If f is large, gain is actually lost.[1]

Using Resistors to Define the Feedback Gain

A convenient way of defining the feedback "gain" (actually loss because it's traditionally less than or equal to one) is by using resistors to form voltage or current division. The easiest amplifier actually has no resistor and is the unity gain amplifier shown in *figure 4*.

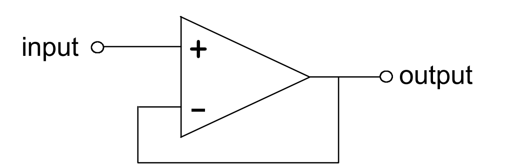

Fig 4: Unity Gain Amplifier

By using resistor voltage division, a positive gain amplifier can be developed as shown in *figure 5*. A similar voltage division technique can be used to design a negative gain amplifier whose goal is to decrease the output until the voltage difference at the inputs of the amplifier is zero, shown in *figure 6*.

[1] Other problems related to large values of f is instability and poor outputs due to the loss of gain which reduces the signal to noise ratio.

Another property of the amplifiers of *figures 5* and *6* is the need of a *ground* or common node predefined to have a value of *0*. But what if the node doesn't happen to zero? What is the gain like then? Writing a system of equations for *figure 5* produces the following:

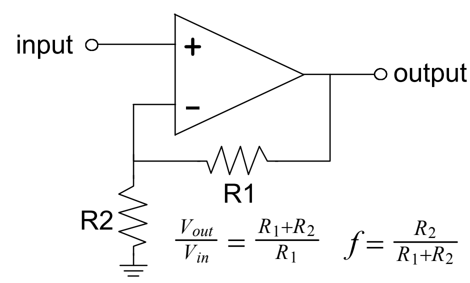

Fig 5: Positive Gain Amplifier

$$V_{out} - V_{gnd} = A \cdot \left((V_{in} - V_{gnd}) - \tfrac{R_2}{R_1+R_2} \cdot (V_{out} - V_{gnd}) \right)$$

$$(V_{out} - V_{gnd}) \cdot \left(1 + A \cdot \tfrac{R_2}{R_1+R_2}\right) = A \cdot (V_{in} - V_{gnd})$$

$$V_{out} \cdot \left(1 + A \cdot \tfrac{R_2}{R_1+R_2}\right) = A \cdot V_{in} + \left(1 + A \cdot \left(\tfrac{R_2}{R_1+R_2} - 1\right)\right) \cdot V_{gnd}$$

$$V_{out} \cdot \left(\tfrac{1}{A} + \tfrac{R_2}{R_1+R_2}\right) = V_{in} + \left(\tfrac{1}{A} + \tfrac{R_2}{R_1+R_2} - 1\right) \cdot V_{gnd}$$

$$\lim_{A \to \infty}: V_{out} \cdot \tfrac{R_2}{R_1+R_2} = V_{in} + \left(\tfrac{R_2}{R_1+R_2} - 1\right) \cdot V_{gnd}$$

$$V_{out} = \tfrac{R_1+R_2}{R_2} \cdot V_{in} + \left(1 - \tfrac{R_1+R_2}{R_2}\right) \cdot V_{gnd}$$

$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{R_1+R_2}{R_2} \text{ and } \frac{\partial V_{out}}{\partial V_{gnd}} = -\frac{R_1}{R_2}$$

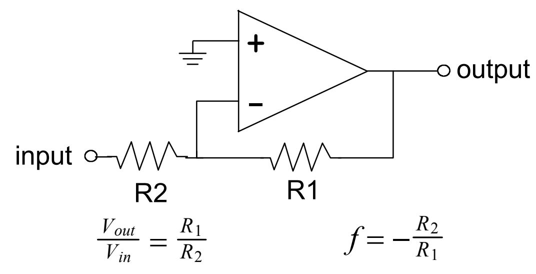

Fig 6: Negative Gain Amplifier

Look! The gain due to moving the ground voltage while keeping the input constant is the same as that for *figure 6*! This shouldn't be too surprising since swapping the input and ground nodes on *figure 5* would give us *figure 6*. What can we do with this if we were to utilize both inputs? We could make a subtractor of sorts with gain... It has an offset of V_{in} which may or may not be desirable.

$$V_{out} = V_{in} + \tfrac{R_1}{R_2} \cdot (V_{in} - V_{gnd})$$

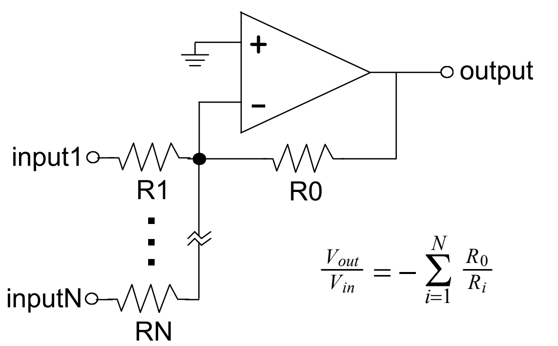

Fig 7: Negative Gain Amplifier/Adder

Is this the only way to make a subtractor? Not exactly, because we can make voltage adders (also with gain) which can be used in combination with negative gain amplifiers for subtraction. The adding amplifier is essentially the negative gain amplifier of *figure 6*, except there are multiple inputs, each with it's own resistor onto the negative input node as shown in *figure 7*.

The easiest way at calculating the gain for the circuit shown in *figure 7* is to sum the currents at the negative input node. Because of feedback, the OpAmp will try to force both positive and negative terminals to be at the same voltage. This effect makes the negative input node become a *virtual ground*, where it can be considered to have the same voltage as the positive input node. This assumption makes the calculations much simpler. This particular configuration can also be seen as a current buffer (to whatever represents the load of R_0) because all of the current flowing through resistors R_1 through R_N must go through R_0 to force the negative input node to be a virtual ground.[2]

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[2] For a more in depth description of this virtual ground and current buffering effect, plus how this circuit configuration can be used as a voltage to curren, current to voltage converter, direct your attention to appendix A at the end of this document.

Using Active Components to Define the Feedback Gain: *Diodes*

Components other than resistors can also be used in feedback loops with more than just linear effects. The diode is an example. Its current is controlled by an exponential relationship with the voltage across it.

$$I_D = I_S \cdot \left(e^{\frac{V_D}{V_T}} - 1\right) \text{ or } V_D \approx V_T \cdot [\ln(I_D) - \ln(I_S)]$$

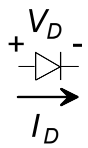
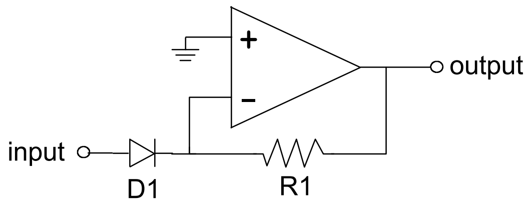


Fig 8: The Diode

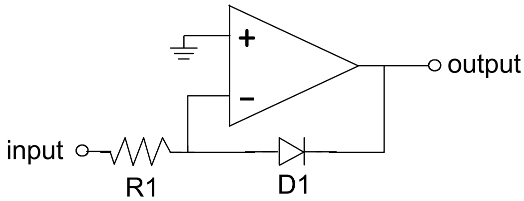
The values of I_S and V_T are, for the most part, constants. I_S is extremely small compared to a typical I_D and V_T is about 26 mV. Remember the inverting amplifier which buffered current? What would happen if the current through a diode was buffered and sent through a resistor? As can be seen in *figure 9*, the output is no longer proportional to the input, but to the log of the input! This has its limitations because the condition $e^{\frac{V_D}{V_T}} \gg 1$ must hold to get decent results out of this, but with exponentials, this can be an easy condition to fulfill...



$$V_{out} = -R_1 \cdot I_S \cdot \left(e^{\frac{V_D}{V_T}} - 1\right)$$

Fig 9: Exponential Gain Amplifier

What if we replaced the other resistor with the diode? The current through the input resistor would be buffered, and the OpAmp would force the output to the voltage needed for the diode to conduct that current. This would result in a logarithmic gain amplifier as shown in *figure 10*.[3] Notice how this essentially reverses the result found in *figure 9*. A possible problem encountered is that I_S is a really small number. This makes a huge positive bias. Fortunately, increasing the size of R_1 can offset this bias with an equally large negative bias. The last, unfortunate, problem with this circuit is the unavoidable fact that logarithmic functions produce low amounts of gain, making the output susceptible to noise.



$$V_{out} = -V_T \cdot [\ln(V_{in}) - \ln(R_1) - \ln(I_s)]$$

Fig 10: Logarithmic Gain Amplifier

Using Active Components to Define the Feedback Gain: *BJTs*

Devices with more than two terminals can be used with feedback to make mathematical functions related to two variables. The BJT is primarily seen as a current driven device which produces current. This makes the current following amplifier described by *figures 6* and *7* the perfect configuration for their use.

$$I_C = I_S \cdot \left(e^{\frac{V_{BE}}{V_T}} - 1\right) \text{ and } V_{BE} \approx V_T \cdot [\ln(I_C) - \ln(I_S)]$$

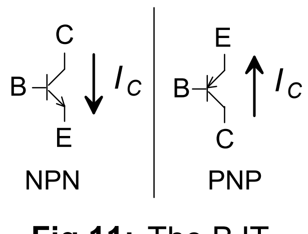


Fig 11: The BJT

Figure 11 shows the schematic symbol for a BJT. The BJT is not really an interesting device at this point because what it does in a feedback loop with OpAmps, we can already do using diodes and OpAmps.

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[3] Because it is a logarithmic function, the input must be positive in the case of *figure 10*. If it isn't, the diode needs to be placed in the reverse direction, and then the input must be negative.

Using Active Components to Define the Feedback Gain: *MOSFETs*

FETs are useful in feedback because they have non-exponential characteristics. The equivalent exponential characteristics of both diodes and BJTs made exploring BJTs uninteresting, however long-channel FETs, in general, follow a square law. This allows us to make circuits that can square values, and take the square root of values. An additional characteristic of FETs in the triode region (aka, linear region) is that they act like adjustable resistors.

A MOSFET has three basic terminals, and sometimes a fourth. They are the gate, source, drain, and bulk (or body). The gate is where the voltage is applied to change the resistivity of the channel between the source and drain. The bulk is what the MOSFET is fabricated on (either a p or n-substrate), and is connected so that it isn't forward biased with either the source and drain (it must be less than or equal to the source and drain voltages for the NMOS and greater than or equal to the source and drain voltages for the PMOS).

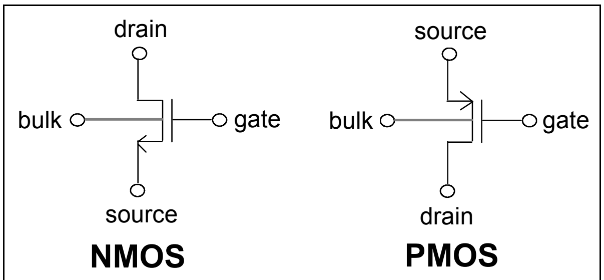

Fig 12: Symbols for MOSFETs

There are three general regions of operation for a MOSFET. They are cutoff, triode, and saturation. Cutoff occurs for voltages differences less than a certain threshold being applied across the gate and source terminals. Triode occurs for when the voltage across the gate and source terminals is larger than the voltage across the source and drain by the cutoff threshold voltage. Saturation is the region where the voltage between the source and drain is large enough to leave the triode region of operation.

For simplicity, V_{GS} is the voltage across the gate and source terminals, V_{DS} is the voltage across the drain and source terminals, and V_{TH} is the cutoff threshold voltage. Using these quantities, we can describe the current flowing through a long-channel MOSFET in the triode region of operation as:

$$I_{DS} = k_n'\left(\frac{W}{L}\right)\left[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

In order for us to treat the MOSFET as a voltage controlled resistor, we'd have to find a relationship where I_{DS} was proportional to V_{DS} to follow Ohm's Law, where $R = \frac{V}{I}$. The only way to do this with the given relationship is to assume $(V_{GS} - V_{TH})V_{DS} \gg \frac{1}{2}V_{DS}^2$. This would allow us to write the equation as:

$$I_{DS} = k_n'\left(\frac{W}{L}\right)(V_{GS} - V_{TH})V_{DS}$$

Where $I_{DS} \propto V_{DS}$. We can then find out the equivalent resistance:

$$\frac{V_{DS}}{I_{DS}} = R_{DS} = \frac{1}{k_n'\left(\frac{W}{L}\right)(V_{GS} - V_{TH})}$$

Notice that this resistance is inversely proportional to $(V_{GS} - V_{TH})$. The only problem is making sure V_{DS} remains small enough compared to $(V_{GS} - V_{TH})$ to keep it operating like a resistor. As we will see later, this puts some constraints on how we design our feedback circuits.

The current flowing through a long-channel MOSFET in the saturation region of operation is described by the equation:

$$I_{DS} = k_n'\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2$$

Notice that this follows a square law.

A Simple Multiplying Stage Using an OpAmp and MOSFET

Using the MOSFET in the triode region of operation in the place of R_2 of *figure 6,* would be perfect for a multiplying amplifier. Given that node X is essentially ground in our model, and choosing a positive voltage for V_{in1}, the source will be at node X and the gate will be some set voltage above this (using a NMOSFET).

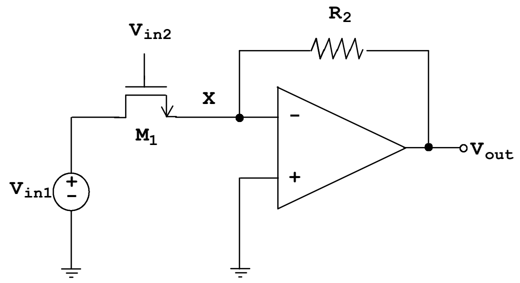

Figure 13 Model of the Feedback Circuit w/ MOS Resistor

Now we must determine what criteria are required to keep the MOSFET in the triode region of operation.

$$V_{\text{in1}} = V_{\text{DS}} \qquad V_{\text{in2}} = V_{\text{GS}}$$

$$(V_{\text{GS}} - V_{\text{TH}})V_{\text{DS}} \gg \tfrac{1}{2}V_{\text{DS}}^2 \qquad R_{\text{DS}} = \frac{1}{k_n'\left(\frac{W}{L}\right)(V_{\text{GS}} - V_{\text{TH}})}$$

$$(V_{\text{GS}} - V_{\text{TH}}) \gg \tfrac{1}{2}V_{\text{DS}} \qquad R_{\text{DS}} \ll \frac{1}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{\text{DS}}}$$

To start having the properties of a variable resister, V_{in2} must be offset by V_{TH}. On top of this, to keep the transistor in the triode region, V_{in2} must be kept greater than V_{in1}. In the end, given the input swing of V_{in1}, the resistance of M_1 is given an upper bound. This upper bound on resistance corresponds to a lower bound on V_{in2}.

Because the gain is $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1}$, having an upper bound on R_1 and a fixed R_2 means there is a lower bound on the gain. A good setup would be to calculate the lowest and highest values needed to be multiplied by the signal, then find out what resistance the highest possible V_{in2} can provide, using this calculate R_2, and then calculate the lowest V_{in2} required. This will give you the range of acceptable values that can be input by V_{in1}, while insuring the types of gains your device requires.[4]

Replacing the resistor R_1 with the equivalent resistance of the MOSFET shows a useful relationship between V_{in1}, V_{in2}, and V_{out}. Let $V_{\text{in2}} = V_{\text{GS}} - V_{\text{TH}}$ to make the relationship easier to see.

$$\frac{V_{\text{out}}}{V_{\text{in1}}} = -\frac{R_2}{R_1} = -k_n'\left(\frac{W}{L}\right)R_2 V_{\text{in2}}$$

$$\frac{V_{\text{out}}}{V_{\text{in1}} \cdot V_{\text{in2}}} = -k_n'\left(\frac{W}{L}\right)R_2$$

This shows that the output voltage is proportional to the product of the two input voltages! Although the product is limited because V_{in2}, cannot be made too small, it may still be used in cases where this is an unnecessary condition.

A Signal Dividing Stage Using an OpAmp

Replacing R_1 with a MOSFET resulted in the product of the input voltages. What if R_1 was replaced instead? Once again, let $V_{\text{in2}} = V_{\text{GS}} - V_{\text{TH}}$ to make the relationship easier to see.

$$\frac{V_{\text{out}}}{V_{\text{in1}}} = -\frac{R_2}{R_1} = \frac{1}{-R_1 k_n'\left(\frac{W}{L}\right)V_{\text{in2}}}$$

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[4] The MOSFET's device characteristics are extremely unpredictable, and cannot be counted on to be the same from device to device. When doing these design steps, this should be taken into consideration, and a reasonable amount of slack should be given to the constraints so your circuit will work even with these variations.

$$\frac{V_{\text{out}}}{\frac{V_{\text{in1}}}{V_{\text{in2}}}} = \frac{1}{-R_1 k_n'\left(\frac{W}{L}\right)}$$

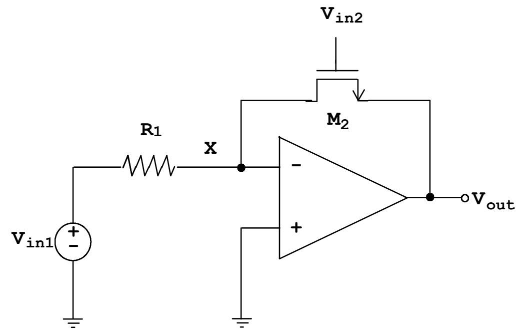


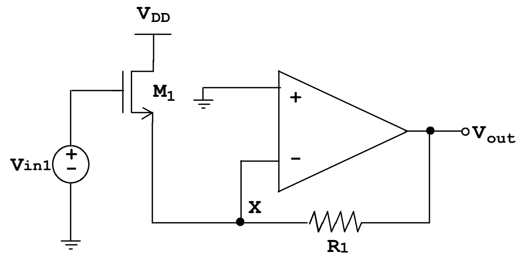
Figure 14 Model of the Feedback Circuit w/ MOS Resistor

Now the output voltage is proportional to the quotient of V_{in1} and V_{in2}.

The same type of criteria for insuring M_1 operated as a resistor in the multiplying stage can be used for insuring M_2 operates as a resistor. The only difference between both cases is that there are now limits on the output voltage swing which will place corresponding limits on the input voltage swing.

A Squaring Amplifier Using an OpAmp

A squaring amplifier can be setup as shown in *figure 15*. Notice how the MOSFET is used to regulate the amount of current let through and the OpAmp is used as simply a current to voltage converter. Notice how the input looses one threshold voltage at the input of the MOSFET. This offset can't be helped, so another offset must be added to whatever signal is squared. Note that this only works while the MOSFET is in saturation, so if V_{DS} falls too low, it will no longer be squaring the input.

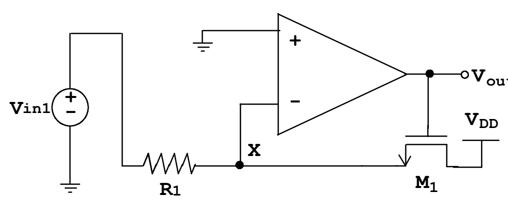


$$V_{out} = -R_1 \cdot k_n'\left(\tfrac{W}{L}\right)(V_{in1} - V_{TH})^2$$

Figure 15 A Squaring OpAmp

A Square Root Amplifier Using an OpAmp

A square root amplifier can be setup as shown in *figure 16*. Notice how the MOSFET is used to regulate the amount of current being allowed to flow through it. If the MOSFET is not on just right, then the node X will not have the voltage required to be a virtual ground. The OpAmp then has to adjust the voltage at the gate to let through, or cutoff current through the path. Notice that the input must be negative for this particular configuration to work. Also notice that the output is offset by a threshold voltage.

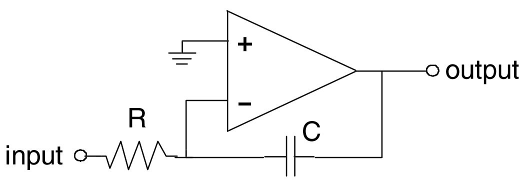


$$V_{out} = V_{TH} - \sqrt{\frac{V_{in1}}{R_1 \cdot k_n'\left(\frac{W}{L}\right)}}$$

Figure 16 A Squaring OpAmp

A Time Integrating Amplifier Using a Capacitor

A capacitor stores charge. If a steady current flows through a capacitor, the charge will build up causing a voltage potential to increase across the capacitor following the formula $Q = CV$ or, rewritten, $V = \frac{Q}{C}$. Using the same current buffer configuration as shown in *figure 6*, a time integrating amplifier can be made without much effort!



$$V_{out} = -\frac{1}{R \cdot C}\int_0^t V_{in}(\tau) \cdot d\tau$$

Fig 17: Integrating Amplifier

This integration can overflow if left integrating for too long (given a non-periodic signal). For this reason, there must be a way to discharge it. One way to discharge it is to put a resistor in parallel with the capacitor. This allows the capacitor to leak by letting some charge to flow around the capacitor. Another way, and probably better, would be to place a MOSFET in parallel with the capacitor. This MOSFET can be opened to discharge the capacitor to a zero value and closed to allow the integrator to function normally.

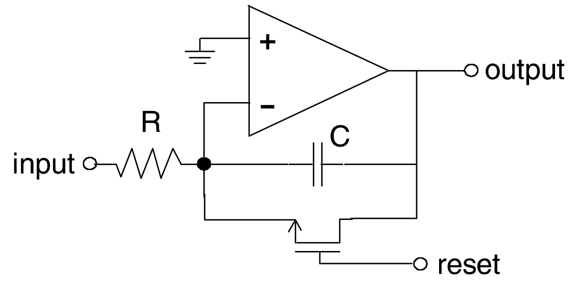
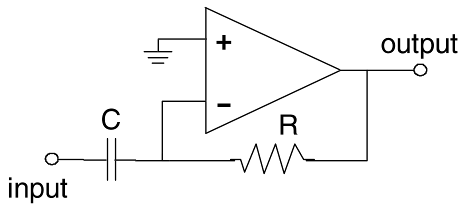


Fig 18: Integrating Amplifier w/ reset

A Time Differentiating Amplifier Using a Capacitor

A capacitor only allows current through if charge moves on or off of it. In this way the change in charge appearing on and off the surface can be detected by sensing the current flowing through it. The current buffer of *figure 6* is once again a perfect way of converting this current into a voltage signal.



$$V_{out} = -R \cdot C \cdot \frac{dV_{in}(\tau)}{d\tau}$$

Fig 19: Differentiating Amplifier

A Peak Detector w/ Integrated Gain

Sometimes you are interested in sampling the highest (or lowest) value within a signal. These can be detected by making the OpAmp only operate in one direction by placing a diode at it's output. A peak detector can be made by using a single diode, however there is a voltage drop that must fall across a diode before a decent amount of current flows through it. To circumvent this problem, an OpAmp can be used to force the output of the diode to match the input by providing current at the output, but when it tries to reverse the value, the diode stops it from operating. Essentially, the current is only allowed to go in one direction, so it charges the output, but won't discharge the output.

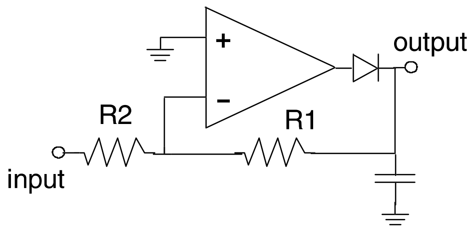


Fig 20: Amplifing Peak Detector

Appendix A : The Virtual Ground

The Current Buffer

As shown in *Figure A1*, if a current source is connected at the input of the negative terminal of the OpAmp, then the current cannot go through the negative terminal, forcing it to flow through whatever else is at the negative terminal (in *Figure A1*, this is a resistor). The current goes through whatever is at that node which has the least amount of resistance. In this way, multiple current sources can all be placed at that node. For the circuit in *Figure A1*, if we know how much current is flowing through *R*, then we also know the voltage across it. If we also know that the OpAmp is in a negative feedback configuration (the type discussed in this article), then the negative terminal will be forced to equal the positive terminal, making the output $V_{out} = V^{+}_{input} + I \cdot R$.

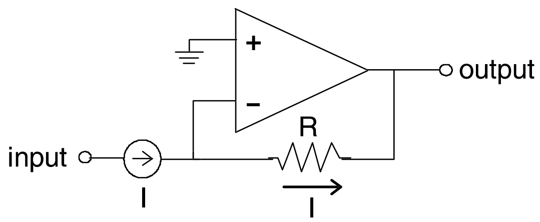


Fig A1: Current Input

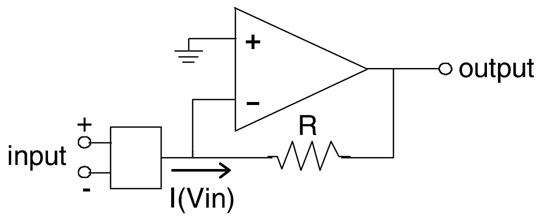


Fig A2: Voltage to Current

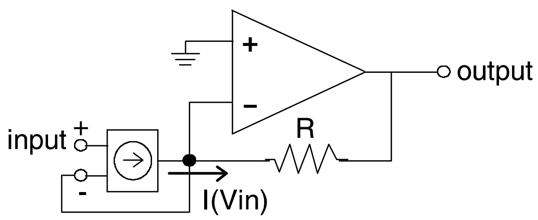


Fig A3: Resistor Type Input Box

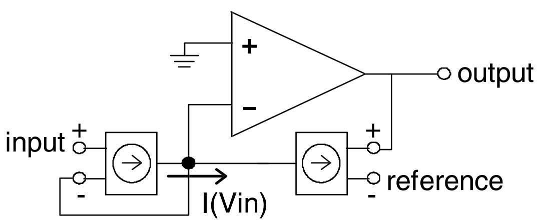


Fig A4: Resistor Type Input Box

Of course, if you have a perfect current source, there is no need to buffer current, because the current source will force the current through the system. This configuration ends up being good for the case where a component, given a voltage input, produces a current output. In the case of a resistor, this voltage input is measured across the device. In some cases, the voltage controlling the current may be measured across something else. A picture of such a device is shown in *Figure A2*. A resistor would be a device which sensed voltage as shown in *Figure A3*.

A similar type construction can be produced for the output of the OpAmp. The OpAmp can control a box which must accept the current from the input box *and* result in the positive and negative OpAmp terminals approaching the same value. If the curent from one box could not flow through the other, a physical law would be broken (current flowing into one point has to go somewhere!), and yet at the same time, the OpAmp works by adjusting the output until both input terminals have the same input voltage (if properly placed in a negative feedback loop). The two types of devices are shown in the figurein the feedback loop of *Figure A4*.

By examining the feedback configuration shown in this article, you can gain some insight into how using the general form shown in *Figure A4* can be used to build new and interesting functions given an input voltage to create an output voltage.